TEMPERATURE STRESSES IN A CYLINDER IN HEAT RESISTANCE TESTS UNDER CONDITIONS OF VARIABLE HEAT TRANSFER

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Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 4, pp. 683–688, 1968

UDC 620.17

The temperature field of a cylinder cooled (heated) in liquid media is numerically determined with allowance for the temperature dependence of the heat transfer coefficient. The results obtained are compared with calculations for α = const. Nonomgrams are given for calculating the maximum thermal stresses in the case of a linear dependence of heat transfer coefficient on temperature.

In most cases modern quantitative heat resistance estimates are based on the quantity $\overline{\Delta T} = \overline{T} - T$. Here, $\overline{T}$ is the cylinder temperature averaged over the section, and T the temperature of the surface (cooling) or central (heating) zone, where the tensile stresses are maximum. When the quasi-static theory of thermoelasticity is employed, this difference uniquely determines the thermal stresses leading to crack initiation and the onset of failure [1, 2].

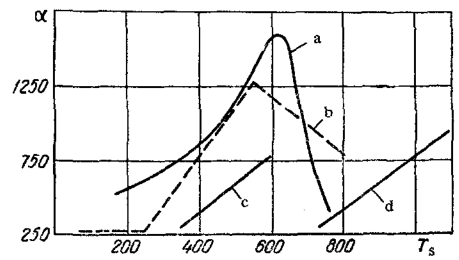

Fig. 1. Heat transfer coefficient (W/m^2 · deg) as a function of temperature (° C) [3]: a) transformer oil; b) approximate curve; c) fused salt, $NaNO_3$; d) fused salt, KCl.

An examination of the experimental methods of determining heat resistance shows that one of the simplest and commonest is the method of cooling in liquid media, which has the advantages of quite high heat transfer coefficients and the small size and simple shape of the specimens.

However, the heat transfer coefficient of the liquid media used for cooling (heating) ordinarily depends on temperature, which leads to nonlinear boundary conditions in calculating the temperature fields. Thus, the heat transfer coefficient of fused salts has a linear temperature dependence; for water, water-oil emulsions, and oil this dependence has a sharply expressed maximum, which may be several times greater than the mean (Fig. 1).

Current methods of estimating the maximum thermal stresses do not take this dependence into account. The calculations are made either with the mean value of the heat-transfer coefficient [4, 5] or the maximum value [6]. The question of the accuracy and limits of applicability of these assumptions remains open.

Existing analytic methods of calculating the temperature field for a variable heat transfer coefficient [7–10] also offer only various approximate methods of estimation, since it is not possible to obtain an exact solution. In practice these methods are inefficient and complicated to employ.

We have calculated numerically the maximum mean temperature difference $\overline{\Delta T}_{max} = (\overline{T} - T_s)_{max}$ of a cylindrical specimen with allowance for the temperature dependence of the heat transfer coefficient using different kinds of coolants.

The heat conduction equation and boundary conditions for an infinitely long cylinder (λ, a = const) cooled in a liquid medium have the form

$$\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial u}{\partial \xi} = \frac{\partial u}{\partial \mathrm{Fo}},$$

$$u(\xi, 0) = 1, \tag{1}$$

$$\left.\frac{\partial u}{\partial \xi}\right|_{\xi=0} = 0,$$

$$\left.\frac{\partial u}{\partial \xi}\right|_{\xi=1} = -\mathrm{Bi}\,(u|_{\xi=1})\,u|_{\xi=1}. \tag{2}$$

For fused salts Bi is a linear function of the dimensionless temperature:

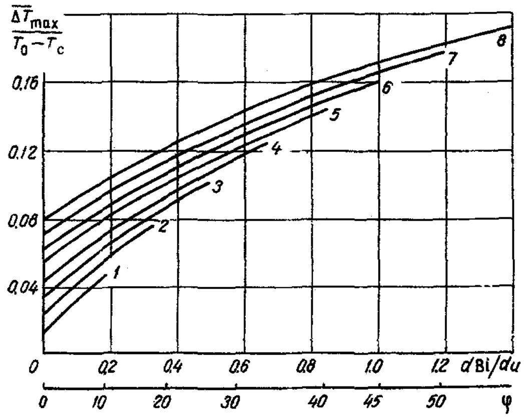

Fig. 2. Maximum dimensionless mean temperature difference for a linear temperature dependence of the heat transfer coefficient (φ = arc tg dBi/du): 1) Bi_0 = 0.05; 2) 0.1; 3) 0.15; 4) 0.2; 5) 0.25; 6) 0.3; 7) 0.35; 8) 0.4.

$$\mathrm{Bi}\,(u) = \frac{d\,\mathrm{Bi}}{du}\,u + \mathrm{Bi}_0, \tag{3}$$

where dBi/du, a constant for a given fused salt, characterizes the slope of the temperature curve of the

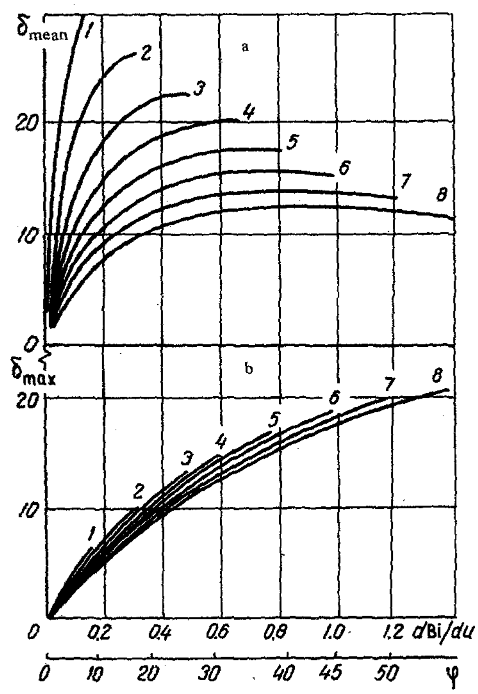

Fig. 3. Relative errors in calculating $\overline{\Delta T}_{max}$ from the mean (a) and maximum (b) values of the heat transfer coefficient as a function of temperature. 1) $Bi_0 = 0.05$; 2) 0.1; 3) 0.15; 4) 0.2; 5) 0.25; 6) 0.3; 7) 0.35; 8) 0.4.

heat transfer coefficient. For coolants such as water and oil we will approximate the Bi(u) relation with a set of linear segments of the form (3) with allowance for the continuity of the Bi(u) curve:

$$\mathrm{Bi}_j(u) = \left.\frac{d\,\mathrm{Bi}}{du}\right|_j u + \mathrm{Bi}_{0j},$$
$$u_j \leqslant u \leqslant u_{j+1}, \tag{4}$$

where u_j and u_{j+1} denote the ends of the j-th segment.

Relative Errors δ in Calculating $\overline{\Delta T}_{max}$ from the Mean and Maximum Values of Bi as a Function of the Initial Temperature Level T_0

T_0, °C	δ_{max}, %			δ_{mean} %		
	$k=0.1$	$k=0.3$	$k=0.6$	$k=0.1$	$k=0.3$	$k=0.6$
800	+32.3	+15.7	+ 3.7	+ 8.1	− 3.2	−11.2
700	+15.4	+ 2.9	+ 3.8	+ 3	− 6.8	− 5.2
600	+ 0.9	+ 9.1	+17.6	− 2.3	+ 5.6	+14.4
500	+17.8	+20.7	+28.3	−24.9	−20.6	−12.2
400	+ 4.9	+11.6	+19.9	−28.5	−21.5	−13.6

The number of segments is selected so as to obtain a given accuracy of approximation of the temperature curve of the heat transfer coefficient; $\overline{\Delta T}_{max}$ was calculated by the pivotal method on an M-20 computer. This method is very convenient for solving problems of this type, since it ensures convergence and the necessary accuracy for an arbitrary relation between the time and coordinate steps.

The calculated values of the dimensionless mean temperature difference at the maximum for a cylinder cooled in a fused salt are presented in Fig. 2. The curve obtained can also serve as nomograms for calculating the maximum thermal stresses when the heat transfer coefficient is a linear function of temperature. Values of the parameters dBi/du and Bi_0 were selected with account for the known temperature dependences of the heat transfer coefficients of fused salts (Fig. 1), the properties of the investigated materials (it was assumed that α lies in the range 300–1000 W/m^2 · deg, λ = 10–30 W/m · deg), and reasonable specimen dimensions (R = 5–15 mm). In the experiment, when the composition of the coolant remains constant, the parameter dBi/du can be varied by changing the initial temperature level and the dimensions of the specimen. The Bi number has its maximum value at the initial moment, i. e., $\mathrm{Bi}_{max} = \mathrm{Bi}(T_0)$, and the mean value lies in the range between the initial temperature and the temperature of the medium, i. e., $\mathrm{Bi}_{mean} = (\mathrm{Bi}_0 + \mathrm{Bi}_{max})/2$. The greatest deviations from the true values $\overline{\Delta T}_{max} \sim$ 20–30% are obtained in calculations based on the mean value at small angles of inclination of the temperature curve Bi(u) (~5–15°) and small levels Bi_0 (0.05–0.15) (Fig. 3). For $\mathrm{Bi}_0 > 0.15$ the error is 10–20%.

Under the same conditions calculations based on the maximum value of Bi give a much smaller error for all Bi_0 (about 5–10%). At medium angles of inclination up to about 45° calculations based on both the maximum and the mean value of Bi give approximately the same error (about 10–20%).

At angles of inclination greater than 45° the error of calculations based on the maximum of Bi, which increases monotonically with the angle of inclination and depends only weakly on the level Bi_0, becomes higher than that for calculations based on the mean; thus $\delta_{max} \geq 20\%$, while δ_{mean} falls to 10–15%.

As already mentioned, liquids such as water, oil, aqueous salt solutions, and water-oil emulsions have a heat transfer coefficient with a sharply expressed maximum, whose magnitude and location vary with the nature and composition of the liquid.

For calculation purposes we selected a typical curve, characteristic of oil, which was approximated by linear segments (Fig. 1):

$$\mathrm{Bi}_j(u) = k\left[\left.\frac{d\alpha}{du}\right|_j u + \alpha_{0j}\right],$$
$$u_j \leqslant u < u_{j+1}, \tag{5}$$

where

$$k = \frac{R}{\lambda} \cdot 10^3 \mathrm{m}^2 \cdot \mathrm{deg/W}.$$

Variation of k makes it possible to use the results of calculations for specimens of different materials with different dimensions and also for any temperature dependence of the heat transfer coefficient similar to that presented in Fig. 1.

The mean and maximum values of Bi characteristic of each segment of the approximate curve were determined as follows. On segment 1 (800–550° C) Bi_{max} was taken equal to Bi (550° C), and $\mathrm{Bi}_{mean} = (\mathrm{Bi}_{in} + \mathrm{Bi}_{max})/2$, where $\mathrm{Bi}_{in} = \mathrm{Bi}(T_0)$ is the initial value of Bi. On segment 2 (550–250° C) Bi_{max} is equal to its initial value, i. e., $\mathrm{Bi}_{max} = \mathrm{Bi}(T_0)$, and $\mathrm{Bi}_{mean} = (\mathrm{Bi}_{max} + \mathrm{Bi}_3)/2$, where Bi_3 is the value of Bi corresponding to segment 3.

We will examine in turn the variation of the errors in determining $\overline{\Delta T}_{max}$ in calculations based on the mean and maximum values of Bi with the initial temperature level (see table).

As the initial temperature level on segment 1 approaches 550° C, the errors of calculations based on Bi_{max} and Bi_{mean} at moderate values of k ~ 0.1–0.3 progressively fall. For an initial level remote from 550° C, the errors in calculating $\overline{\Delta T}_{max}$ from Bi_{max} fall as k increases from 0.1 to 0.6 from about 30 to about 5%, and in calculations based on Bi_{mean} lie within the range ±5–10% for all values of k.

When the initial temperature level varies close to the boundary of segment 1 or on segment 2, while T_m lies within the second segment, the conclusions obtained for fused salts in the range of Bi_0 and dBi/du in question will be perfectly valid. The case when T_m falls within segment 3 is of no special interest, since as the initial temperature level approaches the boundary of the third segment, there is a gradual reduction in the errors involved in calculating $\overline{\Delta T}_{max}$ from both Bi_{mean} and Bi_{max}.

An analysis of the results obtained yields the following conclusions:

1. The errors in calculating $\overline{\Delta T}_{max}$ from the selected heat transfer coefficient (maximum or mean) may reach 20–30% for liquids such as water, oil, or fused salts.

2. By choosing a suitable value of Bi under given cooling conditions when the heat transfer coefficient is a nonlinear function of temperature it is possible to obtain a minimum error in calculating $\overline{\Delta T}_{max}$ of 5–10%. Recommendations concerning the choice of this value of Bi are determined by the results presented above.

3. In the case of a linear temperature dependence of the heat transfer coefficient it is convenient to use the nomograms in Fig. 2 for calculating $\overline{\Delta T}_{max}$.

NOTATION

$u = (T - T_c)/(T_0 - T_c)$ is the dimensionless temperature; $\varepsilon = r/R$ is the dimensionless coordinate; $Fo = a\tau/R^2$ is dimensionless time (Fourier number); $Bi(T) = \alpha(T)R/\lambda$ is the Biot number $Bi_0 = \alpha(T_c)R/\lambda$; T_0 is the initial temperature of cylinder; T_c is the coolant temperature; T_m is the temperature of cylinder surface at maximum of $\overline{\Delta T} = \overline{T} - T_s$.

REFERENCES

1. B. A. Boley and J. H. Weiner, Theory of Thermal Stress [Russian translation], Mir, Moscow, 1964.

2. W. D. Kingery, J. Amer. Ceram. Soc., **38**, no. 1, 1955.

3. A. A. Shmykov, Engineer's Heat Manual [in Russian], Mashgiz, Moscow, 1956.

4. W. B. Crandall and J. Ging, J. Amer. Ceram. Soc., **38**, no. 1, 1955.

5. E. Glenny and M. G. Royston, Trans. Brit. Ceram. Soc., **57**, no. 10, 1958.

6. P. H. Hasselman and W. B. Crandall, J. Amer. Ceram. Soc., **46**, no. 9, 1963.

7. V. V. Ivanov and V. V. Salomatov, IFZh [Journal of Engineering Physics], **9**, no. 1, 1965; **11**, no. 2, 1966.

8. I. M. Prikhod'ko, Izv. VUZ. Aviatsionnaya tekhnika, no. 3, 1963.

9. M. M. Sidlyar, Prikladnaya mekhanika, izd. AN UkrSSR, **1**, no. 7, 1965.

10. V. M. Yudin, Thermal Stresses in Structural Elements [in Russian], izd. AN UkrSSR, Kiev, no. 5, 1965.

7 June 1967